M.Phil. Year 1 Microeconomics 2018-19

Consumer and Producer Theory: Class Exercises for Week 4

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[These questions are taken, with minor adjustments, from past exam papers.]

1. A consumer has utility function

$$u(x_1, x_2) = (1 + x_2)\sqrt{x_1}$$

defined for quantities of two products, 1 and 2. Product 1 can be consumed in continuous quantities (i.e., x_1 can be any non-negative real number), while product 2 is a discrete good (i.e., the possible levels of consumption of product 2 are $x_2 = 0$ or $x_2 = 1$). The consumer's consumption must satisfy her budget constraint $p_1x_1 + p_2x_2 \leq w$, where $p_i > 0$ is the unit price of product $i = 1, 2$ and $w > 0$ is her wealth.

(a) When does the consumer choose to buy product 2?

(b) Is product 1 a necessity or a luxury?

2. A consumer has utility function

$$u(x_1, x_2) = (x_1 + 1)x_2$$

over goods $x_1, x_2 \geq 0$, and faces budget constraint $p_1x_1 + p_2x_2 \leq w$.

(a) Show that the utility function $u(x_1, x_2)$ is strictly quasi-concave.

(b) If $p = (p_1, p_2) = (4, 1)$ and $w = 2$, what is the consumer's demand for x_1 and x_2?

(c) Derive the Walrasian (or Marshallian) demand functions $x_1(p, w)$ and $x_2(p, w)$ for general (p, w).

(d) What is the indirect utility function? Verify Roy's Identity.

3. (a) Suppose there are L products and a consumer's expenditure function takes the Gorman Polar Form:

$$e(p, u) = a(p) + ub(p)$$

Show that the Engel curves are straight lines.

(b) Assume that a consumer's consumption set is $X \subset \mathbb{R}^L$ such that $x_\ell \geq \gamma_\ell$ for each $\ell = 1, \ldots, L$, where $\gamma = (\gamma_1, \ldots, \gamma_L)$ is a vector of parameters with $\gamma_\ell \geq 0$. Suppose that the consumer's utility function defined on X takes the Stone-Geary form:

$$u(x) = \prod_{\ell=1}^{L} (x_\ell - \gamma_\ell)^{\alpha_\ell}$$

where each $\alpha_\ell > 0$ and $\sum_{\ell=1}^{L} \alpha_\ell = 1$. Show that the consumer's expenditure function takes the form in part (a). Interpret $a(p)$ as subsistence expenditure, and $b(p)$ as a price index which represents the marginal cost of living.

4. Consider the constant elasticity of substitution (CES) utility function

$$u(x_1, x_2) = x_1^\theta + x_2^\theta$$

where $0 < \theta < 1$.

(a) Show that u is a quasi-concave function of (x_1, x_2).

(b) Derive the Walrasian (or Marshallian) demand functions $x_1(p, w)$ and $x_2(p, w)$, and the indirect utility function $v(p, w)$. Verify that these functions are homogeneous of degree zero in (p, w).

(c) Show that the elasticity of substitution between goods 1 and 2 is constant and equal to $1/(1 - \theta)$.

Note that the elasticity of substitution between goods 1 and 2 is defined to be

$$\varepsilon(p, w) = -\frac{\partial \left[\frac{x_1(p,w)}{x_2(p,w)}\right]}{\partial \left[\frac{p_1}{p_2}\right]} \frac{\frac{p_1}{p_2}}{\frac{x_1(p,w)}{x_2(p,w)}}$$

5. (This question is about a profit-maximizing firm. However, it could also apply to a utility-maximizing consumer whose level of utility is given.) Let $c(w, q)$ be a firm's minimum cost of producing q units of a single output when input prices are $w = (w_1, \ldots, w_L)$, and let $z(w, q) = (z_1(w, q), \ldots, z_L(w, q))$ be the choice of inputs which minimize its cost of producing this output (so $z(w, q)$ is the conditional factor demand function). Define $s_{ij} = \partial z_i(w, q)/\partial w_j$ for $i, j = 1, \ldots, L$. The $L \times L$ matrix whose (i, j)'th element is s_{ij} is denoted S.

(a) Why is S negative semi-definite and symmetric?

(b) Show that for each $i = 1, \ldots, L$ we have

$$\sum_{j=1}^{L} w_j \frac{\partial z_i(w, q)}{\partial w_j} = 0$$

Deduce that the determinant of S is zero.

(c) The matrix below shows S for a profit-maximizing firm with three inputs at the input prices $w = (1, 2, 6)$:

$$S = \begin{bmatrix} -10 & ? & ? \\ ? & -4 & ? \\ 3 & ? & ? \end{bmatrix}$$

Supply the missing numbers. Does the resulting matrix possess all the properties needed for it to be the result of profit-maximizing behavior?